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MINIMUM COST ROUTES CALCULATION BY MEANS OF DIGITAL ELEVATION MODELS

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RESUMO-NOTAS/ABSTRACT-NOTES

This work deals with the calculation of minimum energy cost routes based on data extracted from a Digital Elevation Model (DEM). The cost is a function of the terrain slope, and path distance. In order to keep the computer (IBM-PC-XT-like) time within reasonable limits, instead of a real terrain, it was used a synthetic model. On a personal workstation a larger set of data can be used. As conclusion it was seen that it is possible to use personal computers for these time consuming calculations, and graphically display the results.

OBSERVAÇÕES/REMARKS

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RESUMO

Este trabalho trata do cálculo de rotas de mínimo custo energético, baseado em dados extraídos de um Modelo Digital de Elevação. O custo é uma função da inclinação do terreno e distância da trajetória. A fim de manter os tempos de computação em limites razoáveis (em um IBM-PC-XT compatível), em vez de dados de um terreno real, foi usado um modelo sintético de terreno. Se for usado uma "workstation" (tipo SUN ou equivalente), uma quantidade maior de dados poderá ser utilizada. Como conclusão, pode se ver que é possível usar computadores pessoais para estes cálculos volumosos, e apresentar os resultados em forma gráfica.

MINIMUM COST ROUTES CALCULATION BY MEANS OF DIGITAL ELEVATION MODELS

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Abstract

This work deals with the calcutation of minimum energy cost routes based on data extracted from a Digital Elevation Model (DEM). The cost is a function of the terrain slope, and path distance. In order to keep the computer (IBM-PC-XT-like) time within reasonable limits, instead of a real terrain, it was used a synthetic model. On a personal workstation a larger set of data can be used. As conclusion it was seen that it is possible to use personal computers for these time consuming calculations, and graphically display the results.

KEY WORDS: Minimum Energy Cost Routes, Digital Elevation Models.

1. INTRODUCTION

The calculation of minimum energy cost routes on a non flat terrain is a problem with important economic implications. For example a large newspaper delivery company operating on a hilly city could save a lot of fuel if their trucks could travel on minimum energy cost routes. The fuel consumption is a function of the slope, and the path distance. These data could be obtained from a Digital Elevation Model (DEM). To simplify the calculations the traffic conditions and the speed could be ignored on a first try. There are also many other problems in witch this technique could be applied. In this work, the idea was to determine if it was possible to solve a problem of this type, and display graphically the solution using a personal computer.

This work uses the mathematical model developed by Black and Pipes (1), for minimum energy cost routes. To simplify the calculations, a synthetic terrain was used, and the energy cost function could take into account only two factors:

a - distance between adjacent points;
b - altitude difference between adjacent points;

A difficult decision is the choice of the energy function dependence on the above factors, since many times some simplifications have to be made. A costly statistical survey is used in general. For the purpose of this work, it was used a study for energetic consumption of pedestrians made by Margaria et al. (2). Other functions could have been used, the software being flexible for their addition.

2. THE MODEL

According to Black and Pipes (1), the minimum energetic cost is based on the minimum calories consumption at a constant and efficient velocity (for a person, walking with a constant velocity not too fast or too slow). This velocity is the one that minimizes the calories consumption per kilometer.

Margaria et al. (2) determined experimentally the energy consumption, in kilocalories per kilogram per kilometer, for an average person under different slopes. This function can be represented as three curves:

- slopes less than -10%, a straight line with the equation:

  y= -0.002*x

- slopes between -10 % and 0%, a parabola represented by the equation:

  y= 0.0032*x*x + 0.062*x + 0.5

- slopes higher than +10 degrees, another straight line with the equation:

  y= 0.07*x +0.5

The minimum energy cost between two adjacent points is then computed by the simple formula:

cost= D*P*y

where D is the distance between the points, P the person's weight, and y the energy consumption. Since P is constant, it will not change the minimum route cost calculation, thus can be made equal to 1, then:

cost=D*y

For each of the DEM points, are computed as many costs as are the neighbouring points - eight for the middle, three or five for the borders. In order to keep the route inside the region of interest the border points are assigned excessive high values. The same trick may be used to represent obstacles, like rivers, lakes, etc.

Using Dinamic Programming, Black and Pipes (1) devised an algorithm to solve this problem as follows:

From "r" as the arrival point, and "i" as the starting point, and if Y(i) is the cost sum from "i" to "r", assuming that optimal implies that the path has a total minimum cost, and if the path starts from "i" to "j", then:

$$Y(i) = Y(j) + C(i,j) \qquad \text{(Eq. 1)}$$

$$Y(i) < Y(k) + C(i,k) \qquad \text{(Eq. 2)}$$

for every "k" different from "j", with C(a,b) being the cost of travelling from a to b.

Equation 1 says that in an optimal path from "i" to "r", whose first movement passes by "j", the total cost is the sum of the costs from an optimal path from "j" to "r" plus the cost of "i" to "j". The next equation, Equation 2, indicates that among all the paths between "i" and "r", none is better than the one that passes by "j".

The Equations 1 and 2 apply to the whole DEM, except at the arrival point "r", then:

$$Y(i) = \min\,[Y(j) + C(i,j)] \qquad \text{(Eq. 3)}$$

for every "i" different from "r", and

$$Y(r) = 0 \qquad \text{(Eq. 4)}$$

A successive approximations method was used to obtain the unknowns, as follows:

$$Y(i)_1 = \min\,[0 + C(i,j)]$$

$$Y(r)_1 = 0$$

the subscript 1 is the iteration number. After n iterations:

$$Y(i)_n = \min\,[Y(j)_{n-1} + C(i,j)]$$

The process ends when, for every $n > n'$, $Y(i)_n - Y(i)_{n'}$ (e, an error as small as reasonable for the problem. The algorithm developed for this work also furnishes the direction that has to be followed, from each pixel to the next in order to minimize the route cost. With the vector of directions to follow, it is possible to draw the minimum cost route, from any starting point "i" to a final destination "r".

3. RESULTS

Starting from the synthetic DEM of Figure 1, with a 81 X 81 pixel grid , a software that runs on DOS on an IBM-PC-XT clone computer was designed, based on the method above. The output can be displayed in two forms: an image in which every pixel corresponds to a cell of the DEM grid, with a color (or grey level) indicating the direction to follow in order to reach the destination point through the minimum cost route, or contour lines of the DEM. In both outputs the minimum cost route can be drawn, as a blue line.

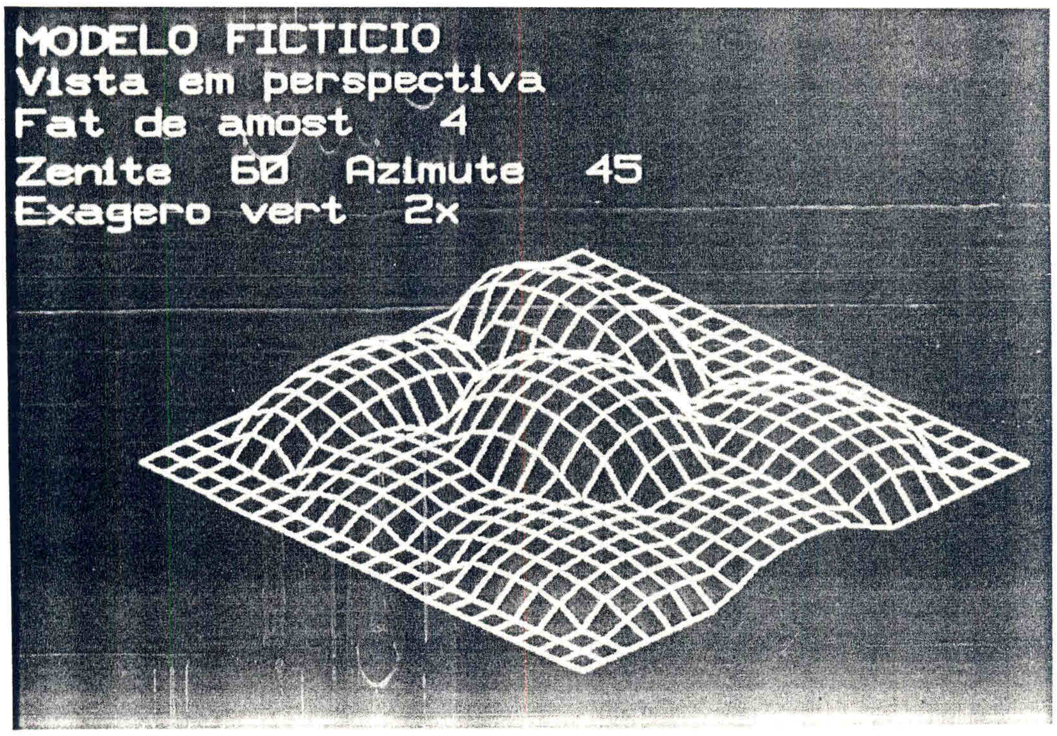

Fig. 1 - Synthetic DEM for the example.

The graphical output was produced by means of a SITIM card, developed at INPE, with a resolution of 512 X 512 pixels (3).

Setting the arrival point at line 50 and column 40, the program has been run, and produced the Figure 2 image that shows the mimimum cost routes from several starting points drawn in blue. It were used the conventions of Table 1 for the directions to follow.

TABLE 1 -- Directions to follow
(North is upwards)

Grey Level	Meaning	Color
0	target	black
1	go to NE	white
2	go to East	grey
3	go to SE	red
4	go to South	dark blue
5	go to SW	green
6	go to West	pink
7	go to NW	yellow
8	go to North	light blue
9	obstacles	dark brown
10	not considered	orange

For better visualization, in Figure 3, it is shown the contour lines for the altitudes of the DEM, with several minimum cost routes drawn in blue. The destination point is the pixel line 50 column 40.

Fig. 2 - Image showing the directions to follow for minimum energy cost routes. Destination point is at position line 50 column 40.

Fig. 3 - Contour line of the DEM with minimum path routes drawn on top. Destination point is line 50 column 40.

4. CONCLUSIONS

The main conclusion that may be arrived at is that it is possible to have implemented a software of this kind on an microcomputer IBM-PC-XT-like. The system performance obtained was not very good, however, with the now available faster machines, the running time can be drastically reduced. The example presented above took about one hour to run. Another test, with a 161 by 161 DEM took seven hours to converge, with 140 iterations. Of course this situation is considered not practical. It should be stated that no attempt was made to optimize the computer code. The authors would like to thanks INPE and IBM Rio Scientific Center for the support, since this work was part of a joint project from both institutions.

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